**Combinatorics IEP HW**

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**1. how many integer numbers from 1 to 10000 are not squares of integers or cubes of integers？**

Let A be the number of squares in the given range, while B is the number of cubes between 1 to 10000. Hence, in order to find the number of integers that are not squares nor cubes, we should find

Where S is the total amount of numbers = 10000. Moreover, since 12, 22, 32, ..., 1002 are all squares in the given range, = 100. In addition, the largest square less than or equal to 10000 would be . Hence, the squares in the given range are 13, 23, 33, ..., 213, giving = 21. Lastly, the number of integers that are both cubes and squares (power of 6) can be calculated as 4.

Therefore, there are 9883 integer numbers from 1 to 10000 that are not squares nor cubes of integers.

**2. How many permutations of 1, 2, 3，……，9 have at least one odd number in its natural position?**

Let A1, A2, A3, A4, and A5 denote the set of permutations that numbers 1, 3, 5, 7, and 9 are in their natural position respectively. Then, since the total number of permutations is 9!, the number of permutations in which at least one of these five odd numbers is in its natural position would be , where since one location is filled with the odd number. Similarly, the total cases could be presented by 1, 2, 3, 4, and all 5 odd numbers being in their natural positions, giving

The number of permutations decreases by one each time an additional odd number is fixed to its natural position, resulting in the number of permutations to go from 8! to 4!.Hence, by calculating the above statement, we would have

Therefore, there are 157824 permutations of 1, 2, 3, ..., 9 in which at least one odd number is in its natural position.

**3. please calculate the number of integral solutions.**

Firstly, in order to have the same lower boundary for all variables, x1 to x4 are replaced with y1 to y4 as follows

giving

Now we can calculate all the non-negative solutions of this equation. Firstly, without any of the upper bounds would have c(13+4-1, 13) = c(16, 13) non-negative solutions. Assuming that A1, A2, A3, and A4 are the solutions when 5respectively, gives

Accordingly

Therefore,

Hence there are 96 integral solutions to this equation.